Matrices

Time : 1 Hour **Maximum Marks: 40**

Compulsory Questions

Q1. If A and B be two non-singular matrices of the same order *n*, then $(AB)^{-1} = B^{-1}A^{-1}$.

Q2. If A is a non-singular matrix of order n, prove that:

(i) $|adj\ A| = |\ A\ |^{n-1}$

(ii) $adj\ (adj\ A) = |\ A\ |^{n-2}\ A$

Q3. Let A and B be invertible matrices of same order *n*. Does $(A + B)^{-1}$ exists? If not, show by an example.

Q4. The diagonal elements of a Skew–Hermitian matrix are either purely imaginary or zero.

Q5. Show that the value of the determinant of a skew-symmetric matrix of odd order is always zero.

OR

Prove that every skew-symmetric matrix of odd order is a singular matrix

Q6. If $A = B + iC$ is Hermitian, show that $A^{\theta}A$ is real if and only if *B* and *C* anti-commute i.e., $BC = -CB$.

Q7. (i) If A is a non-singular symmetric matrices, prove that adj. A is also symmetric.

(ii) If A is a skew-symmetric of order *n*, then show that adj. A is symmetric or skew-symmetric according as *n* is odd or even.

Some Important Question

Q1. If A and B are two matrices of same order *n*, then $adj.(AB) = (adj.B)(adj.A)$

Q2. The necessary and sufficient condition for a square matrix A to be invertible is that A is non-singular (i.e., $|A| \neq 0$)

Q3. If A and B are symmetric matrices; prove that *AB* is symmetric if and only if $AB = BA$.

Q4. Show that every square matrix can be expressed in one and only one way as the sum of a symmetric and skew-symmetric matrices.

Q5. Every square matrix *A* can be expressed in one and only one way as $P + iQ$, where *P* and *Q* are Hermitian matrices.

Q6. Prove that every Hermitian matrix A can be written as $A = B + iC$, where *B* is real and symmetric and *C* is real and skew-symmetric.

Q7. Expressed the matrix $A = \begin{bmatrix} 2 & 1+i & 2+3i \\ 1-i & 1 & -i \\ 2-3i & i & 0 \end{bmatrix}$ in the form $P + iQ$ where P is real and symmetric and *Q* is real and skew-symmetric.

Q8. Show that all positive odd integral powers of a skew-symmetric matrix are skew-symmetric while positive even integral powers are symmetric.

Rank of a Matrix

Time : 1 Hour **Maximum Marks: 40**

Compulsory Questions

Q1. Find the value of *x* so that $\rho(A) \le 2,$ where $A = \begin{bmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{bmatrix}.$

Q2. Reduce the given matrix A to the row-echelon form and hence find the row rank of A. $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 5 & 8 \\ 1 & 3 & 10 & 14 \end{bmatrix}$

Q3. The rank of the product of two matrices cannot exceed the rank of either matrix i.e.,

$\rho(AB) \le \rho(A)$

and $\rho(AB \le \rho(B)$

Q4. Prove that the set of vectors *u* = (1, 3, 2), *v* = (1, –7, –8), *w* = (2, 1, –1) is linearly dependent.

Q5. Prove that if two vectors are linearly dependent, then one of them is the scalar multiple of the other.

Q6. Find *a* if the vectors $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} a \\ 0 \\ 1 \end{bmatrix}$ are linearly dependent.

Q7. Prove that the set of vectors (0, 2, –4) (1, –2, –1). (1, –4, 3) is linearly dependent.

Some Important Question

Q1. Find the condition under which the rank of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & p-2 & 2 \\ 0 & q-1 & p+2 \\ 0 & 0 & 3 \end{bmatrix}$ will be less than 3. Also find that rank.

Q2. Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{bmatrix}; a, b, c$ being all real.

Q3. The rank of matrix remain unaltered by the application of elementary row and column operations.

Q4. Find the rank of the matrix $A=\begin{bmatrix}1 & 2 & -1 & 3\\ -2 & -4 & 4 & -7\\ 1 & 2 & 1 & 2\end{bmatrix}$ by reducing it to normal form.

Q5. Express $A=\begin{bmatrix}1 & 2 & -1\\ 2 & 5 & -2\\ 1 & 2 & 1\end{bmatrix}$ as the product of elementary matrices.

Q6. For the following matrices A, find non-singular matrices P and Q such that PAQ is in normal form and hence determine the rank of A. $A=\begin{bmatrix}1 & 1 & 1\\ 1 & -1 & -1\\ 3 & 1 & 1\end{bmatrix}$

Q7. A square matrix A is singular if and only if its column (rows) are linearly dependent.

Q8. Using elementary operations, find the inverse of the $A=\begin{bmatrix}1 & 2 & -1\\ -1 & 1 & 2\\ 2 & -1 & 1\end{bmatrix}$

Characteristic Equation of a Matrix

Time : 1 Hour **Maximum Marks: 40**

Compulsory Questions

Q1. Find the characteristic roots of the matrix $A = \begin{bmatrix} 5 & 4 & -4 \\ 4 & 5 & -4 \\ -1 & -1 & 2 \end{bmatrix}$.

Q2. Prove that 0 is latent root of a matrix if and only if A is singular.

Q3. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, use Cayley-Hamilton theorem to express $2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial in A.

Q4. Prove that the matrix A and $B^{-1}AB$ have the same latent roots (characteristic roots); B being an invertible matrix of the same order as A.

Q5. If α is an eigen value of a non-singular matrix A, then prove that $\frac{|A|}{\alpha}$ is an eigen value of adj. A.

Q6. Let A be a 3 × 3 upper triangular matrix with real entries. If $a_{11} = 1, a_{22} = 2$ and $a_{33} = 3,$ determine α, β, γ such that $A^{-1} = \alpha A^2 + \beta A + \gamma I$.

Q7. Show that every identity matrix of order $n \geq 2$ is derogatory.

Some Important Questions

Q1. Find the eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

Q2. Determine the value of *a*, *b* and *c* so that (1, 0, –1) and (0, 1, –1) are eigen vectors of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ a & 3 & 2 \\ 3 & b & c \end{bmatrix}$

Q3. Every square matrix satisfies its characteristic equation.

Q4. Find the characteristic equation of the matrix $A=\begin{bmatrix}1 & 1 & 3\\ 5 & 2 & 6\\ -2 & -1 & -3\end{bmatrix}$ and show that A satisfies this equations.

Q5. Prove that the characteristic roots of the Hermitian matrix are all real.

Q6. Prove that the characteristic roots of a Skew-Hermitian matrix are either zero or purely imaginary.

Q7. Find the minimal polynomial of the matrix $A=\begin{bmatrix}5 & -6 & -6\\ -1 & 4 & 2\\ 3 & -6 & -4\end{bmatrix}$ and show that it is derogatory.

Q8. Show that $\begin{bmatrix}2 & 3 & 4\\ 0 & 2 & -1\\ 0 & 0 & 1\end{bmatrix}$ is non-derogatory and obtain its minimal equation.

Application of Matrices to a System of Linear Equations

Time : 1 Hour **Maximum Marks: 40**

Some Important Questions

Q1. Solve : $x + y + 2z + w = 5$

$$2x + 3y - z - 2w = 2$$

$$4x + 5y + 3z = 7$$

Q2. For what value of γ, does the system $\begin{bmatrix} -1 & 2 & 1 \\ 3 & -1 & 2 \\ 0 & 1 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ has

(i) no solution (ii) unique solution.

Q3. For what value of λ, the equations

$$x + y + z = 1$$
$$x + 2y + 4z = \gamma$$
$$x + 4y + 10z = \lambda^2.$$

Q4. Find the values of *a* and *b* for which the following system of linear equations

$$2x + by - z = 3$$
$$5x + 7y + z = 7$$
$$ax + y + 3z = a$$

Q5. For what values of λ μ the system of equations:

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

Has (i) no solution, (i) a unique solution, (iii) an infinite number of solutions.

Q6. For what value of λ will be equations:

$$3x - y + \lambda z = 1$$
$$2x + y + z = 2$$
$$x + 2y - \lambda z = -1$$

Fail to have a unique solution? Will the equation have any solution for this value of λ?

Q7. Solve:

$$x-2y+z-w=0$$
$$x+y-2z+3w=0$$
$$4x+y-5z+8w=0$$
$$5x-7y+2z-w=0$$

Q8. Find the value of such that the system of equations:

$$x+ky+3z=0$$
$$4x+3y+kz=0 \qquad \text{has a non trivial solution}$$
$$2x+y+2z=0$$

Q9. Find the value of k such that the following system of equations has a non-trivial solution:

$$(3x-8)x+3y+3z=0$$
$$3x+(3x-8)y+3z=0$$
$$3x+3y+(3x-8)z=0$$

Q10. Show that the only real value of λ for which the equations

$$x+2y+3z=\lambda x$$
$$3x+y+2z=\lambda y$$
$$2x+3y+z=\lambda z$$

Have a non-zero solution is 6.

Orthogonal and Unitary Matrices

Time : 1 Hour **Maximum Marks: 40**

Compulsory Questions

Q1. The determinant of an orthogonal matrix is ≠ 1.

OR

Every orthogonal matrix is non-singular.

Q2. The transpose of a unitary matrix is unitary.

Q3. The determinant of a unitary matrix has absolute value 1.

Q4. Show that the matrix $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is orthogonal if and only if $A=\begin{bmatrix} a & b \\ -b & a \end{bmatrix} or \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$, where $a^2+b^2=1$.

Q5. Show that if A is Hermitian and P is unitary, then $P^{-1}AP$ is Hermitian.

Q6. If A is unitary and Hermitian matrix, then show that A is involutory (i.e., $A^2 = 1$)

Q7. If A is unitary matrix of order *n*, then show that it is non-singular and | A | = ≠ 1.

Some Important Question

Q1. The inverse and transpose of an orthogonal matrix are orthogonal.

Q2. Product of two unitary matrices is unitary.

Q3. Prove that the characteristic roots of a unitary matrix are of unit modulus.

OR

Prove that the absolute value of each characteristic root of unitary matrix is unity.

Q4. If A is a real skew-symmetric matrix such that $A^2 + I = O$, show that A is orthogonal and is of even order.

Q5. If *A* is Skew-Hermitian and $(A - I)$ is non-singular, show that $P = (A + I)(A - I)^{-1}$ is unitary.

Q6. If $(l_r, m_r; n_r), r = 1, 2, 3$ be the direction cosines of three mutually perpendicular lines referred to an orthogonal Cartesian co-ordinate system, then prove that $\begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$ is an orthogonal matrix.

Q7. Prove that if AB = BA and C is an orthogonal matrix, then the multiplication of C′AC and C′BC is commutative.

Q8. If A is unitary and B = AP, where P is non-singular, then show that PB^{-1} is unitary.

Relation Between the Root and Coefficients of an Equation

Time : 1 Hour **Maximum Marks: 40**

Compulsory Questions

Q1. Given that –4 is a root of the equation $2x^3+6x^2+7x+60=0$. Find the other roots.

Q2. If ax^3+bx+c has a factor of the form $^2+\lambda x+1,$ show that $a^2-c^2=ab.$

Q3. Form an equations with rational co-efficient two of whose roots are 1 + 5*i* and 5 – *i*.

Q4. If *b* and *c* are real and $2-\sqrt{-3}$ is a root of the equation $x^3+x^2+bx+c=0,$ what are the other roots and what is the value of *c*?

Q5. If the roots of the equation $x^n-1=0$ be 1, $\alpha,\beta,\gamma,.......,$ show that $(1-\alpha)(1-\beta)(1-\gamma)....=n.$

Q6. Solve the equation $x^4-8x^3+14x^2+8x-15=0,$ roots being in A.P.

Q7. Find the condition that the roots of the equation $x^3+3px^2+3qx+r=0$ are in (i) A.P. (ii) G.P.

Q8. Find the condition that two roots of the equation $x^3-bx^2+cx-d=0$ be equal.

Some Important Question

Q1. Find the remainder in the division of $x^3+3px+q$ by $(x-a)^2$ and deduce that $x^3+3px+q=0$ has two equal roots if $q^2+4p^3=0.$

Q2. Use the method of synthetic division to express $f(x)=5x^4-3x^3+x^2+x+1$ as a polynomial in powers of (*x* –1).

Q3. Find the values of *a* and *b* so that (*x* –3) may exactly divide the expression $2x^4-7x^3+ax+b.$

Q4. Every equation $f(x)=0$ of the nth degree has *n* roots, and no more.

Q5. In an equation with real co-efficient, imaginary roots occur in conjugate pairs.

Q6. Solve the equation $x^4+15x^3+70x^2+120x+64=0,$ whose roots are given to be in G.P.

Q7. Find the condition that sum of two roots of the equation $x^4-px^3+qx^2-rx+s=0$ may be equal to the sum of the other two.

Q8. Solve the equation $x^4-9x^2+4x+12=0,$ given that it has a multiple root.

Transformation of Equations

Time : 1 Hour **Maximum Marks: 40**

Compulsory Questions

Q1. Remove the fractional co-efficients from the equation $x^4+\frac{1}{2}x^3-\frac{5}{3}x^2+\frac{2}{3}x-1=0.$

Q2. If the roots of the equation $x^3-ax^2+bx-c=0$ are in harmonic progression, show that the mean root is $3c/b$.

Q3. Find the equation whose roots exceed by 2 the roots of the equation $4x^4+32x^3+83x^2+76x+21=0$. Hence solve the equation.

Q4. If α,β,γ are the roots of the equation $x^3+px^2+qx+r=0;$ form an equation whose roots are $\alpha-\frac{1}{\beta\gamma},\ \beta-\frac{1}{\gamma\alpha},\ \gamma-\frac{1}{\alpha\beta}.$

Q5. If $\alpha,\beta,\ \gamma$ are the roots of $x^3-x-1,$ show by transforming technique that $\frac{1+\alpha}{1-\alpha}+\frac{1+\beta}{1-\beta}+\frac{1+\gamma}{1-\gamma}=-7$

Q6. If α,β,γ are the roots of the equation $x^3+ax^2+bx+c=0,$ form an equation whose roots are $\frac{\beta+\gamma}{\alpha},\ \frac{\gamma+\alpha}{\beta},\frac{\alpha+\beta}{\gamma}.$

Q7. Find the equation whose roots are the squares of the roots of $x^3+qx+r=0.$

Some Important Question

Q1. Solve the equation $15x^4-8x^3-14x^2+8x-1=0,$ given that the roots are in H.P.

Q2. Show that the same transformation can remove both second and fourth terms of the equation $x^4+16x^3+83x^2+152x+84=0$ and hence solve it.

Q3. The difference between two roots of the equation $2x^3+x^2-7x-6=0$ is 3. Solve it by diminishing the roots by 3.

Q4. Transform the equation $x^4-4x^3-18x^2-3x+2=0$ into one which is wanting in the third term.

Q5. If α,β,γ are the roots of the cubic $x^3+3x+2=0,$ find the equation whose roots are $(\alpha-\beta)(\alpha-\gamma),(\beta-\gamma)(\beta-\alpha),(\gamma-\alpha)(\gamma-\beta).$ Hence show that the given cubic has two imaginary roots.

Q6. If the roots of the equation $x^3-6x^2+11x-6=0$ be α, β, γ: find the equation whose roots are $\beta^2+\gamma^2,\gamma^2+\alpha^2,\alpha^2+\beta^2.$

Q7. Find the equation of squared differences of the cubic $x^3+6x^2+2=0$.

Q8. Find the equation of squared differences of the roots of the equation $x^3+3x+2=0$ and show that the given equation has a pair of imaginary roots.

Solution of Cubic and Biquadratic Equations, Descarte's Rule of Signs

Time : 1 Hour **Maximum Marks: 40**

Compulsory Questions

Q1. Show that the equation $x^8 + 5x^3 + 2x - 3 = 0$ has atleast six imaginary roots.

Q2. Show that for all values of c, the equation $x^5 + 5x^2 + 3x + c = 0$ has atleast two imaginary roots.

Q3. Show that the equation $x^{2n} - 1 = 0$ has only two real roots.

Q4. Show that the equation $x^7 - 3x^4 + 2x^3 - 1$ has atleast four complex roots.

Q5. Show that the equation $x^{12} - 2x^4 + x^3 - 3x^3 + 12 = 0$ must have atleast six imaginary roots.

Q6. Show that the equation $x^7 + x^4 + 8x + k = 0$ has atleast four imaginary roots for all values of *k*.

Q7. Show that the equation $2x^7 + 3x^4 + 3x + k = 0$ has atleast 4 imaginary roots for al values of k (constant).

Some Important Question

Q1. To discuss the nature of the roots of the cubic $Z^3 + 3HZ + G = 0$.

Q2. Solve the equation $28x^3 - 9x^2 + 1 = 0$ by Cardan's method.

Q3. Show that the roots of equation $x^3 - 3x + 1 = 0$ are $2\cos\frac{2\pi}{9}, 2\cos\frac{8\pi}{9}, 2\cos\frac{14\pi}{9}$.

Q4. Solve the equation $x^3 + 3x - 14 = 0$ by Cardan's method.

Q5. Solve by the method of resolution into quadratic factors $x^4 - 2x^3 - 5x^2 + 10x - 3 = 0$

Q6. Solve the equation $x^4 - 6x^3 + 8x^2 + 2x - 1 = 0$ by Descarte's method.

Q7. Solve $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$ by Ferrari's method.

Q8. Solve the equation $2x^4 + 6x^3 - 3x^2 + 2 = 0$ by Ferrari's method.